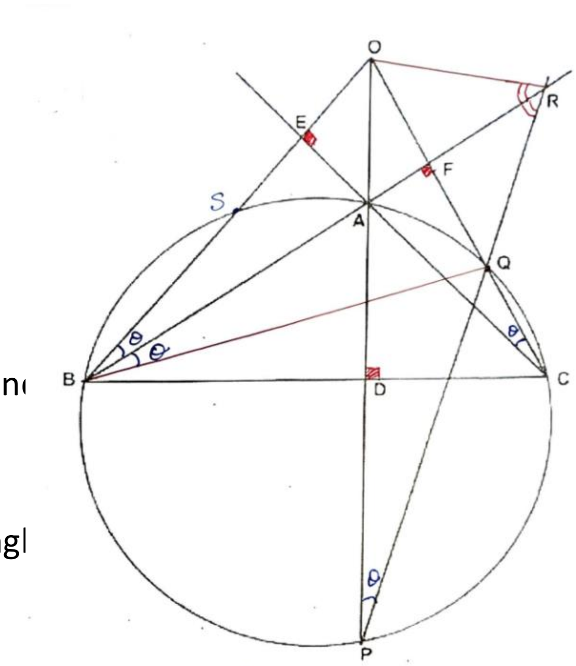
01.05.2023 – I Prize Cash Award Winner Solution



To Prove: $OB^2 = AB \times BR$

Construction: Join OR, BQ

Proof :

In the given figure as $\angle D = \angle E = \angle F = 90°$

$\Longrightarrow$ Quadrilaterals AECD, AFCD, AEOF are cyclic an

Quadrilaterals BEFC is also cyclic.

Let $\angle EBA = \theta \Longrightarrow \angle QCA = \theta$ [Same segment angles]

$\Longrightarrow \angle QCA = \angle QPA = \theta$ & [Same Segment angl

$\angle QPA = \angle QBA = \theta$

Consider $\Delta\, OBF$ & $\Delta\, QBF$

$\angle B = \angle B = \theta$

BF = BF [common side]

$\angle F = \angle F = 90°$ [Given]

$\therefore \Delta OBF \cong \Delta QBF$ [ASA congruency]

$\Longrightarrow$ OF = FQ [CPCT]

New consider Δ ROF & Δ RQF

OF = FQ [Proved]

$\angle F = \angle F = 90°$ [Given]

RF = RF [Common side]

∴ Δ ROF ≅ Δ RQF [SAS Congruency]

$\angle ORF = \angle QRF$ [CPCT] --------------(1)

Now consider Δ APR & Δ OBR

$\angle P = \angle B = \theta$

$\angle ARP = \angle ORB$ [from (1)]

∴ Δ APR ~ΔOBR [AA similarity]

$\Longrightarrow \frac{AP}{OB} = \frac{PR}{BR} = \frac{AR}{OR}$ [Proportionality of sides]

$\frac{BR}{OB} = \frac{PR}{AP}$ -------------------(2)

Now consider ΔOBA & ΔRPA

$\angle B = \angle P = \theta$ [from figure]

$\angle OAB = \angle RAP$ [V. Opp Angles]

∴ΔOBA $\sim \Delta RPA$ [AA similarity]

$\therefore \frac{OB}{RP} = \frac{BA}{PA} = \frac{OA}{RA}$ [Proportionality of sides]

$\frac{OB}{AB} = \frac{PR}{PA}$ ------------------------(3)

From (2) & (3)

$\frac{BR}{OB} = \frac{OB}{AB}$

$\Longrightarrow OB^2 = AB \times BR$ ------------- Hence Proved.

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